# Determination of the Molecular Packing in the Crystal of Coumarin by Means of Potential-Energy Calculations 

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#### Abstract

Coumarin ( $o$-coumaric acid lactone, $\mathrm{C}_{9} \mathrm{H}_{6} \mathrm{O}_{2}$ ) forms orthorhombic crystals, space group Pca2,$Z=4$, $a=15 \cdot 466$ (12), $b=5 \cdot 676$ ( 6 ), $c=7.917$ (6) $\AA$. The structure was solved by potential-energy calculations coupled with the minimum residual analysis. Three-dimensional X-ray data were measured with an offline Siemens automatic single-crystal diffractometer by the $\omega$-scan technique. The refinement was carried out by least-squares methods and the final $R$ is 0.048 . The carbon and oxygen atoms of coumarin lie in the same plane, the greatest deviation being $0.015 \AA$. The crystal packing is characterized mainly by van der Waals and dipole-dipole interactions. However, the dipole-dipole contribution to the total potential energy does not seem to determine the molecular packing.


## Introduction

Van der Waals, hydrogen bonding, coulombic and ion-dipole semi-empirical potentials have been verified in known crystal structures and employed to solve the phase problem (Dosi, Giglio, Pavel \& Quagliata, 1973; Coiro, Giglio, Lucano \& Puliti, 1973, and references quoted therein). Another energy term, which may be important in the stabilization of the crystal lattice, is that arising from the interactions among permanent electric dipoles. In order to ascertain the role that the dipole-dipole energy plays in determining the energy minimum, which corresponds to the experimental packing, the crystal of coumarin has been examined. Coumarin is a rigid molecule (Fig. 1) which possesses a large dipole moment in the range 3.8-4.9 D (Jatkar \& Deshpande, 1960). Since it is probable that several nearly equal van der Waals minima exist in the packing analysis of an aromatic molecule, especially if translational degrees of freedom are present in the crystal, it is possible that it is the dipole-dipole energy that is conclusive in locating the actual minimum. On the other hand Kitaigorodskii \& Mirskaya (1965) have demonstrated that the dipole-dipole contribution to the total potential energy is small.
A further point of interest concerns the determination of the hydrogen atom positions in coumarin in order to compare distances between hydrogen atoms with those deduced from n.m.r. experiments on oriented molecules in a nematic mesophase (Cappelli, Di Nola \& Segre, 1974). Although X-ray analysis is not very suitable for this purpose, the accurate location of the heavier atoms may lead to rather precise coor-
dinates for the hydrogen atoms. After this work was completed we became aware of the crystal structure determination of coumarin by Miasnikova, Davydova \& Simonov (1973), hereinafter referred to as MDS.

## Experimental

Crystals of coumarin (Merck-Schuchardt) were grown from ethyl ether in the form of colourless rhombohedral pyramids, m.p. $68^{\circ} \mathrm{C}$. They are orthorhombic and the unit-cell dimensions, as measured on a diffractometer with Mo $K \alpha$ radiation ( $\hat{\lambda}=0.7107 \AA$ ) at room temperature, are: $a=15.466$ (12), $b=5.676$ (6), $c=7.917$ (6) $\AA$. A density of $1.41 \mathrm{~g} \mathrm{~cm}^{-3}$, calculated for four molecules per unit cell, agrees with the value of 1.39 (1) $\mathrm{g} \mathrm{cm}^{-3}$ measured by flotation in an aqueous solution of $\mathrm{Cd}\left(\mathrm{NO}_{3}\right)_{2}$.

The space groups Pca2 $2_{1}$ or Pcam were indicated by the systematic absences $h 0 l$ and $0 k l$ with $h$ and $l$ odd respectively. However, the space group Pcam can be discarded because it would require the molecule to lie in a mirror plane at $c=\frac{1}{4}$, resulting in a systematic decrease in the $00 l$ structure factors with increasing $l$. An inspection of the intensities shows that this is not true. A crystal of dimensions $1.4 \times 0.4 \times 0.5 \mathrm{~mm}(\mu=$ $1.08 \mathrm{~cm}^{-1}$ ) was mounted on the Siemens automatic single-crystal diffractometer AED, equipped with a scintillation counter and pulse-height analyser, the $c$ axis being nearly coincident with the polar $\varphi$ axis of the goniostat. The crystal was sealed in a glass capillary since coumarin sublimes very readily. Intensities were recorded at room temperature with zirconiumfiltered Mo $K \alpha$ radiation. The setting angles $\chi, \varphi$ and
$\theta$ were accurately measured for 14 reflexions with a very narrow counter aperture and were used for a leastsquares refinement of orientation. The reflexions were recorded up to $2 \theta=58^{\circ}$, each reflexion being scanned twice at a rate of $2.5^{\circ} \mathrm{min}^{-1}$ over a range from -0.75 to $0.75^{\circ}$. The moving-crystal stationary-detector ( $\omega$-scan) technique was employed. Background counts were taken for a time equal to that of the scan.
The intensities of 750 independent reflexions greater than $3 \sigma(I)$ out of a possible total of 938 were collected. Four standard reflexions, measured after every 40 reflexions, remained essentially constant throughout the run, showing only the deviations from the mean predicted by counting statistics. The data were corrected first for counting losses (Arndt \& Willis, 1966) with $t=2.5 \times 10^{-6} \mathrm{~s}$ and $K=1$ and then for background. Standard deviations were assigned according to the formula $\sigma(I)=\left[P+B+(p I)^{2}\right]^{1 / 2}$ where $P$ is the total integrated peak count obtained in the two scans, $B$ is the total background count, $I=P-B$ and $p$ is the 'ignorance factor' (Corfield, Doedens \& Ibers, 1967) fixed as 0.06 .

## Molecular packing determination

The molecular packing was determined by means of potential-energy calculations (Coiro, Giglio \& Quagliata, 1972). The packing energy in the crystal depends on three rotations ( $\psi_{1}, \psi_{2}, \psi_{3}$ ) and two translations ( $t_{x}, t_{y}$ ) along the $a$ and $b$ axes. The rotations were performed in a right-handed orthogonal framework $O x y z$, coinciding with the crystallographic system Oabc. $\psi_{1}$, $\psi_{2}$ and $\psi_{3}$ stand for clockwise rotations about $O z, O x$ and $O z$, provided that they are accomplished in the order given, moving the orthogonal system and keeping the molecule fixed.
The starting model of the coumarin molecule was taken to be planar with bond lengths $\mathrm{C}-\mathrm{C}=1 \cdot 40, \mathrm{C}=\mathrm{O}$ $=1.24, \mathrm{C}-\mathrm{O}=1.37$ and $\mathrm{C}-\mathrm{H}=1.08 \AA$, and trigonal bond angles, except for the following: $\mathrm{C}(2)-\mathrm{O}(1)-\mathrm{C}(9)$ $=124 \cdot 50^{\circ} ; \mathrm{O}(11)-\mathrm{C}(2)-\mathrm{O}(1)=120 \cdot 25^{\circ} ; \mathrm{C}(3)-\mathrm{C}(2)-\mathrm{O}(1)$ $=\mathrm{C}(10)-\mathrm{C}(9)-\mathrm{O}(1)=117.75^{\circ} ; \quad \mathrm{O}(11)-\mathrm{C}(2)-\mathrm{C}(3)=$ $122.00^{\circ}$. The atomic coordinates in $\AA$ of the molecule at $\psi_{1}=\psi_{2}=\psi_{3}=0^{\circ}$ and $t_{x}=t_{\nu}=0 \AA$ were generated (Gavuzzo, Pagliuca, Pavel \& Quagliata, 1972) and are reported in Table 1. The coefficients of the van der Waals potentials in the generalized form:

$$
V(r)=a \underset{r^{d}}{a \exp }(-b r)-c r^{-6}
$$

are listed in Table 2. Angular and translational increments of $20^{\circ}$ and $0.5 \AA$ were given in the first run, assuming a cut-off distance of $7.5 \AA$. The scanning of the multi-dimensional parametric space was performed in the reduced Cheshire cell of Hirshfeld (1968) taking into account both the space group and the molecular symmetry.

The van der Waals energy was estimated by considering all the intermolecular non-bonded interac-

Table 1. Atomic fractional coordinates in the starting position

|  | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{O}(1)$ | -0.0864 | 0.0000 | 0.1534 |
| C(2) | -0.0452 | 0.0000 | $0 \cdot 3066$ |
| $\mathrm{C}(3)$ | 0.0452 | 0.0000 | 0.3066 |
| C(4) | 0.0904 | 0.0000 | 0.1533 |
| C(5) | 0.0904 | 0.0000 | $-0.1533$ |
| C(6) | 0.0452 | 0.0000 | -0.3066 |
| C(7) | -0.0452 | 0.0000 | -0.3066 |
| C(8) | -0.0904 | 0.0000 | -0.1533 |
| $\mathrm{C}(9)$ | -0.0452 | 0.0000 | 0.0000 |
| $\mathrm{C}(10)$ | 0.0452 | 0.0000 | 0.0000 |
| $\mathrm{O}(11)$ | -0.0876 | 0.0000 | 0.4395 |
| H(3) | 0.0801 | 0.0000 | 0.4248 |
| $\mathrm{H}(4)$ | 0.1601 | 0.0000 | 0.1533 |
| H(5) | 0.1601 | 0.0000 | -0.1533 |
| H(6) | 0.0801 | 0.0000 | -0.4248 |
| H(7) | $-0.0801$ | 0.0000 | -0.4248 |
| H(8) | -0.1601 | 0.0000 | -0.1533 |

Table 2. The coefficients of the van der Waals pctential functions used in the packing-energy calculations
The energy is in kcal per atom pair if the interatomic distance is in $\AA$.

| Interaction | $a\left(\times 10^{-3}\right)$ | $b$ | $c$ | $d$ |
| :---: | :---: | :---: | :---: | ---: |
| H-H | $6 \cdot 6$ | $4 \cdot 080$ | $49 \cdot 2$ | 0 |
| H-C | $44 \cdot 8$ | $2 \cdot 040$ | $125 \cdot 0$ | 6 |
| H-O | $42 \cdot 0$ | $2 \cdot 040$ | $132 \cdot 7$ | 6 |
| C-C | $301 \cdot 2$ | $0 \cdot 000$ | $327 \cdot 2$ | 12 |
| C-O | $278 \cdot 7$ | $0 \cdot 000$ | $342 \cdot 3$ | 12 |
| O-O | $259 \cdot 0$ | $0 \cdot 000$ | $358 \cdot 0$ | 12 |

tions between 17 atoms of one molecule and the atoms of the nearest 13 molecules with multiplicity 2 . The regions of minima were subsequently explored by decreasing the angular and translational increments finally to $0.5^{\circ}$ and $0.0125 \AA$, by a program, written by C. Quagliata, in the following manner. Each parameter to be optimized is changed either positively or negatively by a prespecified amount. If a successful change occurs the parameter assumes the value corresponding to the lowest energy (otherwise it is left unchanged) and the same procedure is then repeated for the other parameters. The new set of parameters is again tested as indicated above and another set is determined. This process continues until the energy improvement for an entire cycle is less than a prespecified amount $E$. Thereafter the increments and $E$ are halved and the same process applied anew. This analysis is continued until it fails to decrease the energy more than an amount fixed at the beginning of the program. As an option the minimum search may be interrupted if the increments of the parameters become smaller than some values previously established. Two minima noticeably deeper than the others were found with parameters:

$$
\begin{array}{r}
\text { (I) } 23 \cdot 5,108 \cdot 0,76 \cdot 0^{\circ} ; 5 \cdot 36,1 \cdot 22 \AA \text {; } \\
\text { (II) } 25 \cdot 5,109 \cdot 0,83 \cdot 0^{\circ} ; 1 \cdot 70,1 \cdot 34 \AA .
\end{array}
$$

Four parameters are almost the same, while $t_{x}$ differs by about $a / 4$. Their energy values are nearly equal, being separated by only 0.2 kcal . These two minima were tested by computing $R$ for the 52 reflexions with $\sin \theta / \lambda \leq 0.25 \AA^{-1}$ as a function of the five rotational and translational degrees of freedom (Damiani, Giglio, Liquori \& Ripamonti, 1967). The best $R$ values were 0.22 and 0.10 for (I) and (II) respectively and it was therefore decided to start the refinement of (II).

## Refinement of the structure

The programs of Domenicano, Spagna \& Vaciago (1969) were used to refine the structure and for the geometrical calculations. Atomic scattering factors for carbon and oxygen were taken from Cromer \& Mann (1968) and for hydrogen from Hanson, Herman, Lea \& Skillman (1964).

Refinement proceeded by isotropic and anisotropic block-diagonal least-squares methods and was carried out with only the 750 observed reflexions. The function minimized is $\sum w\left(\left|\left|F_{o}\right|-\left|F_{c}\right|\right|\right)^{2}, w$ being equal to $1 / \sigma^{2}\left(F_{o}\right)$. A difference synthesis showed all the hydrogen atoms at reasonable positions, on peaks between 0.20 and $0.37 \mathrm{e}^{-3}$. The hydrogen atoms were included in the last three cycles with the same isotropic $B$ of the carbon atom to which each is linked. Their positional parameters were held fixed during the refinement. The refinement was considered to be complete when the parameter shifts were less than $50 \%$ of the estimated standard deviations. The final $R$ and weighted $R$ were 0.048 and 0.071 respectively for the observed reflexions. The final atomic coordinates and anisotropic thermal parameters are reported in Tables 3 and 4 with their standard deviations. The bond


Fig. 1. Bond distances with estimated standard deviations in brackets. The estimated standard deviations of the $\mathrm{C}-\mathrm{H}$ bonds are omitted.

Table 4. Final values of the thermal parameters with their standard deviations in parentheses
The form of the anisotropic temperature factor is $\exp \left[-10^{-4}\left(b_{11} h^{2}+b_{12} h k+b_{13} h l+b_{22} k^{2}+b_{23} k l+b_{33} l^{2}\right)\right]$.

|  | $b_{11}$ | $b_{12}$ | $b_{13}$ | $b_{22}$ | $b_{23}$ | $b_{33}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}(1)$ | 49 (1) | 2 (4) | -1 (3) | 263 (6) | 108 (10) | 162 (3) |
| C(2) | 45 (1) | -30(7) | 14 (4) | 322 (9) | - 50 (14) | 145 (4) |
| C(3) | 39 (1) | 28 (6) | 6 (5) | 357 (10) | -18(15) | 171 (5) |
| C(4) | 45 (1) | 36 (6) | -5 (5) | 283 (8) | 8 (13) | 150 (5) |
| C(5) | 51 (1) | -40 (6) | 3 (5) | 320 (9) | 50 (15) | 146 (5) |
| C(6) | 57 (2) | -72 (8) | 14 (6) | 447 (15) | 63 (18) | 153 (5) |
| C(7) | 41 (1) | -5 (8) | 6 (5) | 473 (13) | -67 (17) | 171 (5) |
| C(8) | 48 (1) | 26 (7) | -18(5) | 335 (10) | 18 (14) | 155 (5) |
| C(9) | 44 (1) | -16(5) | -8(4) | 216 (7) | -9 (10) | 112 (4) |
| C(10) | 41 (1) | 2 (5) | -15 (4) | 215 (7) | -37(10) | 126 (4) |
| O(11) | 55 (1) | -76 (5) | 47 (6) | 416 (8) | 41 (14) | 245 (5) |
| H(3) | 4.0 |  |  |  |  |  |
| H(4) | $3 \cdot 7$ |  |  |  |  |  |
| H(5) | $4 \cdot 1$ |  |  |  |  |  |
| H(6) | $4 \cdot 7$ |  |  |  |  |  |
| H(7) | 4.6 |  |  |  |  |  |
| H(8) | $4 \cdot 0$ |  |  |  |  |  |

lengths and angles are given in Figs. 1 and 2 with the standard deviations in parentheses. A list of observed and calculated structure factors together with the phase angles is given in Table 5 .

## Discussion

The crystal structure of coumarin was solved by MDS from the Patterson function. The refinement was carried out by the block-diagonal least-squares method with isotropic temperature parameters. Only 539 independent reflexions were collected and the final $R$ was $0 \cdot 104$ for the observed reflexions. An attempt to refine the structure anisotropically gave unreliable bond lengths.
Although Professor Kitaigorodskii informed us about the work of MDS before the draft of the manuscript of our paper was finished, we believe that our results deserve to be published for the following reasons:
(1) We have determined the molecular packing by applying a new method based on potential-energy cal-
culations. The results are of interest for the development of the method.
(2) We have used more reflexions and our anisotropic refinement was successful. MDS ascribe the failure of their anisotropic refinement to the lack of an absorption correction. However the same situation was found by us when intensities collected from a crystal not sealed in a glass capillary were employed, and is no doubt due to the rapid sublimation.

The agreement between our results and those of MDS is satisfactory. The differences between the molecular geometries are summarized in Table 6. The greatest discrepancies involve the atoms $C(6)$ and $C(7)$, which have some of the highest isotropic as well as anisotropic temperature factors. The molecular geometry can be compared with that of 4 -hydroxycoumarin monohydrate (Gaultier \& Hauw, 1966) and of 3-bromo-4-hydroxycoumarin (Gaultier \& Hauw, 1965). However, this last compound yields less accurate bond distances and angles owing to the presence of the bromine atom. The C-C distances in the $\mathrm{C}(5)-\mathrm{C}(10)$ ring lie between $1 \cdot 36_{8}$ and $1 \cdot 39_{5} \AA$ (average $1 \cdot 38_{3} \AA$ )

Table 5. Observed structure amplitudes and calculated structure factors


Table 5 (cont.)

and are somewhat below normal. This shortening may be due to thermal libration. The double-bond charact $\_$r for $\mathrm{C}(3)-\mathrm{C}(4)$ appears to be retained as for 4-hydroxycoumarin ( $1.34_{4}$ and $1.35 \AA$ respectively). Furthermore the $\mathrm{C}(2)-\mathrm{C}(3)$ and $\mathrm{C}(4)-\mathrm{C}(10)$ bonds, which are adjacent to the double bond, are systematically longer than $1 \cdot 40 \AA$ in both compounds. The bonds $\mathrm{C}(2)-\mathrm{O}(1)$ and $\mathrm{C}(9)-\mathrm{O}(1)$ are nearly equal (about $1 \cdot 37$

Table 6. Differences between the bond lengths and angles of this work and of MDS

The difference is calculated assigning a minus sign to the value reported by MDS.

| $C(2)-O(11)$ | $-0.012 \AA$ | $C(3)-C(2)-O(11)$ | $-0 \cdot 4^{\circ}$ |
| :--- | ---: | :--- | ---: |
| $C(2)-O(1)$ | 0.001 | $O(1)--C(2)-O(11)$ | $1 \cdot 1$ |
| $C(9)-O(1)$ | -0.001 | $C(3)-C(2)-O(1)$ | $-0 \cdot 6$ |
| $C(8)-C(9)$ | $-0 \cdot 007$ | $C(2)-O(1)-C(9)$ | $0 \cdot 1$ |
| $C(7)-C(8)$ | $0 \cdot 010$ | $C(10)-C(9)-O(1)$ | $0 \cdot 0$ |
| $C(6)-C(7)$ | $-0 \cdot 034$ | $C(8)-C(9)-O(1)$ | $-0 \cdot 1$ |
| $C(5)-C(6)$ | $-0 \cdot 008$ | $C(8)-C(9)-C(10)$ | $0 \cdot 2$ |
| $C(5)-C(10)$ | $0 \cdot 000$ | $C(7)-C(8)-C(9)$ | $-1 \cdot 4$ |
| $C(9)-C(10)$ | $0 \cdot 003$ | $C(6)-C(7)-C(8)$ | $1 \cdot 7$ |
| $C(4)-C(10)$ | $-0 \cdot 020$ | $C(5)-C(6)-C(7)$ | $-0 \cdot 3$ |
| $C(3)-C(4)$ | $0 \cdot 007$ | $C(6)-C(5)-C(10)$ | $0 \cdot 2$ |
| $C(2)-C(3)$ | $0 \cdot 010$ | $C(5)-C(10)-C(9)$ | $-0 \cdot 3$ |
|  |  | $C(4)-C(10)-C(5)$ | $-0 \cdot 2$ |
|  |  | $C(4)-C(10)-C(9)$ | $0 \cdot 5$ |
|  |  | $C(3)-C(4)-C(10)$ | $-0 \cdot 1$ |
|  |  | $C(2)-C(3)-C(4)$ | $0 \cdot 2$ |

$\AA$ ) except in the case of 3-bromo-4-hydroxycoumarin ( 1.31 and $1.43 \AA$ ) where the estimate of these bond lengths is less reliable.

The $C(7)-C(8)-C(9)$ bond angle is smaller than $120^{\circ}$ in all the compounds. The angles $\mathrm{C}(8)-\mathrm{C}(9)-\mathrm{O}(1)$ and $\mathrm{C}(4)-\mathrm{C}(10)-\mathrm{C}(5)$ at the junction of the two rings are respectively smaller and greater than $120^{\circ}$ in these three molecules and cause the approach of $\mathrm{O}(1)$ to $C(8)$ and the removal of $C(4)$ from $C(5)$. The angles around $\mathrm{C}(2)$ in coumarin (Fig. 2) agree with those of 4-hydroxycoumarin reported in parentheses: $125.6^{\circ}$ $\left(125^{\circ}\right), 117 \cdot 3^{\circ}\left(117^{\circ}\right)$ and $117 \cdot 2^{\circ}\left(117^{\circ}\right)$. This geometry resembles that of a carboxylic group. The leastsquares plane passing through the carbon and oxygen atoms, all considered with unit weight, is:

$$
2.5363 x+2.4824 y+6.7287 z-0.9969=0 .
$$

This equation is referred to the crystal axes and $x, y$ and $z$ are fractional coordinates. The atomic deviations from the least-squares plane are reported in Table 7. The molecule is planar, as is 4-hydroxycouma-


Fig. 2. Bond angles with estimated standard deviations in brackets. The estimated standard deviations of the $\mathrm{C}-\mathrm{C}-\mathrm{H}$ angles are omitted.


Fig. 3. View of the packing along b. The circles indicate the oxygen atoms.
rin, the greatest deviation being $0.015 \AA$. The molecular packing is shown in Fig. 3. All intermolecular approaches agree well with the normal van der Waals separations. The shortest distances for each type of atom-pair are: $\mathrm{H} \cdots \mathrm{H}=2 \cdot 87, \mathrm{H} \cdots \mathrm{C}=2 \cdot 97, \mathrm{H} \cdots \mathrm{O}=$ $2 \cdot 60, \mathrm{C} \cdots \mathrm{C}=3 \cdot 43, \mathrm{C} \cdots \mathrm{O}=3 \cdot 37, \mathrm{O} \cdots \mathrm{O}=3 \cdot 42 \AA$, using the coordinates of the hydrogen atoms found in the difference synthesis. The oxygen atom $\mathrm{O}(11)$ is involved in many intermolecular contacts and is probably the atom which contributes most to the stabilization of the crystal lattice by means of van der Waals interactions.

Table 7. Atomic deviations ( $\AA$ ) from the least-squares plane

| $\mathrm{O}(1)$ | -0.007 | $\mathrm{C}(5)$ | -0.012 | $\mathrm{C}(9)$ | -0.009 |
| :--- | ---: | ---: | ---: | :--- | ---: |
| $\mathrm{C}(2)$ | -0.007 | $\mathrm{C}(6)$ | -0.002 | $\mathrm{C}(10)$ | 0.007 |
| $\mathrm{C}(3)$ | -0.007 | $\mathrm{C}(7)$ | 0.007 | $\mathrm{O}(11)$ | 0.007 |
| $\mathrm{C}(4)$ | 0.015 | $\mathrm{C}(8)$ | 0.006 |  |  |

There are several types of van der Waals interactions, all equally important. Nevertheless the energy minimum corresponding to the actual structure has been located with reasonable accuracy. This is a satisfactory test of the qualitative reliability of the potentials employed in the packing analysis.

Coumarin oriented in a nematic mesophase has been studied by n.m.r. (Capelli, Di Nola \& Segre, 1974). The intramolecular $\mathrm{H}-\mathrm{H}$ distances were determined on the basis of the dipole-dipole coupling constants, assuming the $\mathrm{H}(4)-\mathrm{H}(5)$ distance to be $2.49 \AA$ as in deuterated naphthalene (Pawley \& Yeats, 1969). Since the hydrogen atoms cannot be located with sufficient accuracy by X-ray diffraction methods we have preferred to generate them on the bisectors of the $\mathrm{C}-\mathrm{C}-\mathrm{C}$ angles at the expected positions ( $\mathrm{C}-\mathrm{H}=1.08 \AA$ ) in order to verify the n.m.r. results. The n.m.r. and X-ray results (Table 8) are in good agreement.

Table 8. Comparison between n.m.r. and X-ray intramolecular $\mathrm{H}-\mathrm{H}$ distances $(\AA)$

|  | N.m.r. | X-ray |
| :--- | :---: | :---: |
| $\mathrm{H}(1)-\mathrm{H}(2)$ | $2 \cdot 47$ | $2 \cdot 41$ |
| $\mathrm{H}(1)-\mathrm{H}(3)$ | $4 \cdot 80$ | $4 \cdot 78$ |
| $\mathrm{H}(1)-\mathrm{H}(4)$ | $6 \cdot 73$ | $6 \cdot 70$ |
| $\mathrm{H}(1)-\mathrm{H}(5)$ | $7 \cdot 10$ | $7 \cdot 08$ |
| $\mathrm{H}(1)-\mathrm{H}(6)$ | $5 \cdot 77$ | $5 \cdot 82$ |
| $\mathrm{H}(2)-\mathrm{H}(3)$ | $2 \cdot 54$ | $2 \cdot 57$ |
| $\mathrm{H}(2)-\mathrm{H}(4)$ | $4 \cdot 84$ | $4 \cdot 83$ |
| $\mathrm{H}(2)-\mathrm{H}(5)$ | $5 \cdot 96$ | $5 \cdot 92$ |
| $\mathrm{H}(2)-\mathrm{H}(6)$ | $5 \cdot 53$ | $5 \cdot 53$ |
| $\mathrm{H}(3)-\mathrm{H}(4)$ | $2 \cdot 49$ | $2 \cdot 44$ |
| $\mathrm{H}(3)-\mathrm{H}(5)$ | $4 \cdot 31$ | $4 \cdot 23$ |
| $\mathrm{H}(3)-\mathrm{H}(6)$ | $4 \cdot 97$ | 4.94 |
| $\mathrm{H}(4)-\mathrm{H}(5)$ | $2 \cdot 49$ | $2 \cdot 44$ |
| $\mathrm{H}(4)-\mathrm{H}(6)$ | $4 \cdot 31$ | $4 \cdot 28$ |
| $\mathrm{H}(5)-\mathrm{H}(6)$ | 2.49 | 2.48 |

This agreement, which would be even better if the $\mathrm{H}(4)-\mathrm{H}(5)$ distance were slightly decreased, shows the possibility of applying n.m.r. spectroscopy for direct resolution with high precision to structural problems concerning the determination of the coordinates of hydrogen atoms. At present, unfortunately, this analysis can only be carried out on relatively simple molecules containing at most eight hydrogen atoms not related by symmetry (Diehl \& Khetrapal, 1969).

The dipole-dipole energy was calculated from

$$
\begin{array}{r}
V\left(r_{a b}, \theta_{a}, \theta_{b}, \varphi\right)=-14 \cdot 393 \mu_{a} \mu_{b} r_{a b}^{-3}\left(2 \cos \theta_{a} \cos \theta_{b}\right. \\
\left.-\sin \theta_{a} \sin \theta_{b} \cos \varphi\right)
\end{array}
$$

where $\mu_{a}$ and $\mu_{b}$ represent the electric moments of two point dipoles $r_{a b}$ apart, $\theta_{a}$ and $\theta_{b}$ are the angles formed by the vectors of magnitude $\mu_{a}$ and $\mu_{b}$ respectively with $r_{a b}$, and $\varphi$ is the dihedral angle between the planes passing through $\mu_{a}, r_{a b}$ and $\mu_{b}, r_{a b}$. V is in kcal if $\mu_{a}$ and $\mu_{b}$ are in Debye units and $r_{a b}$ in $\AA$.

The induced effects, which are generally less important, were neglected. A dipole moment of 4.5 D was assigned to the coumarin molecule and the corresponding vector was oriented in the molecular plane on a straight line passing approximately through the centroid of the $\mathrm{C}(2)-\mathrm{O}(1)$ ring and intersecting the $\mathrm{C}(2)-\mathrm{O}(1)$ bond in such a way as to make an angle of about $15^{\circ}$ with the axis of the carbonyl group (Le Fèvre \& Le Fèvre, 1937; Jatkar \& Deshpande, 1960). In additional calculations the direction of the molecular dipole moment was varied over a large range. In fact the dipole-dipole energy did not reach the best value near the actual packing. Moreover this energy is strongly dependent on the orientation and on the application point of the dipole moment vector.

Therefore we were not able to narrow the range of energy values relative to the experimental structure. However it is clear that the dipole-dipole energy term is unnecessary to establish the molecular packing, as for $5 \alpha$-androstan-3,17-dione (Coiro, Giglio, Lucano \& Puliti, 1973).

If this hypothesis is supported by other evidence the tedious computation of this energy term, not always feasible because many molecular dipole moments and their directions have been not determined, can be neglected.

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## References

Arndt, U. W. \& Willis, B. T. M. (1966). Single Crystal Diffractometry, p. 144. Cambridge Univ. Press.
Cappelli, E., Di Nola, A. \& Segre, A. L. (1974). Mol. Phys. In the press.

Coiro, V. M., Giglio, E., Lucano, A. \& Puliti, R. (1973). Acta Cryst. B29, 1404-1409.
Coiro, V. M., Giglio, E. \& Quagliata, C. (1972). Acta Cryst. B28, 3601-3605.
Corfield, P. W. R., Doedens, R. J. \& Ibers, J. A. (1967). Inorg. Chem. 6, 197-204.
Cromer, D. T. \& Mann, J. B. (1968). Acta Cryst. A24, 321-324.
Damiani, A., Giglio, E., Liquori, A. M. \& Ripamonti, A. (1967). Acta Cryst. 23, 681-687.

Diehl, P. \& Khetrapal, C. L. (1969). NMR Basic Principles and Progress, Vol. 1. New York: Springer-Verlag.
Domenicano, A., Spagna, R. \& Vaciago, A. (1969). Atti Accad. Nazl. Lincei Rend. Classe Sci. Fis. Mat. Nat. 47, 331-336.
Dosi, C., Giglio, E., Pavel, V. \& Quagliata, C. (1973). Acta Cryst. A 29, 644-650.

Gaultier, J. \& Hauw, C. (1965). Acta Cryst. 19, 927-933.
Gaultier, J. \& Hauw, C. (1966). Acta Cryst. 20, 646-651.
Gavuzzo, E., Pagliuca, S., Pavel, V. \& Quagliata, C. (1972). Acta Cryst. B28, 1968-1969.

Hanson, H. P., Herman, F., Lea, J. D. \& Skillman, S. (1964). Acta Cryst. 17, 1040-1044.

Hirshfeld, F. L. (1968). Acta Cryst. A 24, 301-311.
Jatkar, S. K. K. \& Deshpande, C. M. (1960). J. Indian Chem. Soc. 37, 19-24.
Kitaigorodski, A. I. \& Mirskaya, K. V. (1965). Sov. Phys. Cristallogr. 9, 539-544.
Le Fèvre, C. G. \& Le Fèvre, R. J. W. (1937). J. Chem. Soc. pp. 1088-1090.
Miasnikova, R. M., Davydova, T. C. \& Simonov, V. I. (1973). Kristallografiya, 18, 720-724.

Pawley, G. S. \& Yeats, E. A. (1969). Acta Cryst. B25, 2009-2013.

# The Crystal and Molecular Structure of 2,3-exo-Ferroco-4,4-dimethylbicyclo[3,2,1]octa-2,6-diene, a Product of the Reaction of Cyclopentadiene with the 2-Ferrocenyl-2-propyl Cation 

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(Received 3 December 1973; accepted 2 January 1974)
$\left(\pi-\mathrm{C}_{5} \mathrm{H}_{5}\right) \mathrm{Fe}\left(\pi-\mathrm{C}_{13} \mathrm{H}_{15}\right)$ crystallizes in space group $P 2_{1} / c, a=10.733, b=8 \cdot 122, c=16.873 \AA, \beta=110.95^{\circ}$, with four formula units in the unit cell. The crystals are reflexion twins about the (001) face and the structure was determined with X-ray data from one half of the twin and refined by the least-squares method to $R=0.091$ for 842 independent reflexions. The substituted cyclopentadienyl moiety had the predicted tricyclic configuration formed with a bridging ethylene group. The cyclopentadienyl rings are in the eclipsed conformation and inclined at an angle of $7^{\circ}$.

## Introduction

Turbitt \& Watts (1974) reported that the ferrocenyl carbonium ion (I) reacts stereospecifically with cyclopentadiene to give a single product for which the structure (II) was proposed from ${ }^{1} \mathrm{H}$ n.m.r. evidence.

The crystal structure of this product has been determined in order to confirm the proposed structure and to establish its stereochemical features. The results have been reported in a preliminary communication (Cameron, Maguire, Turbitt \& Watts 1973).

(I)

(II)

## Experimental

The crystals were prepared by Turbitt \& Watts (1974).

## Crystal data

$\mathrm{C}_{18} \mathrm{H}_{20} \mathrm{Fe}$, M.W. 519.14, $F(000)=616$.
Monoclinic: $a=10.733 \pm 0.003, b=8.122 \pm 0.001, c=$ $16.873 \pm 0.005 \AA, \beta=110.95 \pm 0.01^{\circ}$.
$D_{m}=1.414, D_{c}=1.412 \mathrm{~g} \mathrm{~cm}^{-3}$, Мо $K \alpha \lambda=0.7107 \AA$, $\mu=10.15 \mathrm{~cm}^{-1}$.
Systematic extinctions $h 0 l: l=2 n+1,0 k 0: k=2 n+1$; space group $P 2_{1} / c\left(C_{2 h}^{5}\right.$, No. 14).

The crystals are thin orange plates and all appear to be reflexion twins about the $(001)$ face which is parallel to the face of the plates. It was not possible to bisect any crystal and retain a single crystal large enough for X-ray intensity measurements. A twinned crystal was mounted about the unique axis and the intensities of 924 independent reflexions (with $I>2 \sigma$ )

